ISRO EC-2017: Solutions of Control Systems

1. The system with the transfer function $\frac{Y(s)}{X(s)} = \frac{s}{s+p}$ has an output $y(t) = \cos\left(2t - \frac{\pi}{3}\right)$ for the input signal $x(t) = p\cos\left(2t - \frac{\pi}{2}\right)$. Then, the system parameter p is

(a) $\sqrt{3}$
(b) 1
(c) $2/\sqrt{3}$
(d) $\sqrt{3}/2$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s}{s+p} \; ; \quad y(t) = \cos\left[2t - \frac{\pi}{3}\right].$$

$$x(t) = p\cos\left[2t - \frac{\pi}{2}\right].$$

$$x(t) \longrightarrow \boxed{\text{System}}_{H(s)} \longrightarrow y(t) = B\cos\left[\omega_0 t + \phi_2\right]$$

$x(t) = A\cos\left[\omega_0 t + \phi_1\right]$

$$\phi_1 = -\frac{\pi}{2}, \quad \phi_2 = -\frac{\pi}{3}$$

$$\begin{cases} B = A \cdot |H(j\omega_0)| \\ \phi_2 = \phi_1 + \angle H(j\omega_0) \end{cases}$$

$$\omega_0 = 2$$

$$H(j\omega) = \frac{j\omega}{j\omega + p} \; ; \quad H(j\omega_0) = \frac{j2}{j2+p}$$

$$\angle H(j\omega_0) = 90^\circ - \tan^{-1}\frac{2}{p}$$

$$+\frac{\pi}{3} = -\cancel{\frac{\pi}{2}} + \cancel{90^\circ} + \tan^{-1}\frac{2}{p} \Rightarrow \frac{2}{p} = \tan\frac{\pi}{3} = \sqrt{3}$$

$$\boxed{p = \frac{2}{\sqrt{3}}}$$

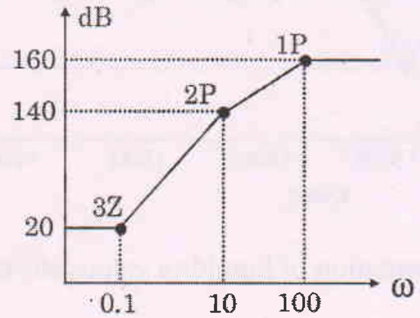
option c ✓

2. The approximate Bode magnitude plot of a minimum–phase system is shown in the figure below. The transfer function of the system is

$\omega_{z_1} = .1 \to T_{z_1} = \frac{1}{.1}$

$\omega_{p_1} = 10 \to T_{p_1} = \frac{1}{10}$

$\omega_{p_2} = 100 \to T_{p_2} = \frac{1}{100}$



(a) $10^8 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$

(b) $10^7 \frac{(s+0.1)^3}{(s+10)^2(s+100)}$

(c) $10^8 \frac{(s+0.1)^3}{(s+10)^2(s+100)}$

(d) $10^7 \frac{(s+0.1)^2}{(s+10)^2(s+100)}$

MPS → all zeros poles → lies in LHS

$$T(s) = \frac{K(sT_{z_1}+1)\cdots}{s^r(sT_{p_1}+1)\cdots} = \frac{10\left(s\frac{1}{.1}+1\right)^3}{s^0\left(s\frac{1}{10}+1\right)^2\left(s\frac{1}{100}+1\right)}$$

Initial slope $= -20\frac{db}{dec} \times r \Rightarrow \boxed{r=0} \Rightarrow$ type zero system

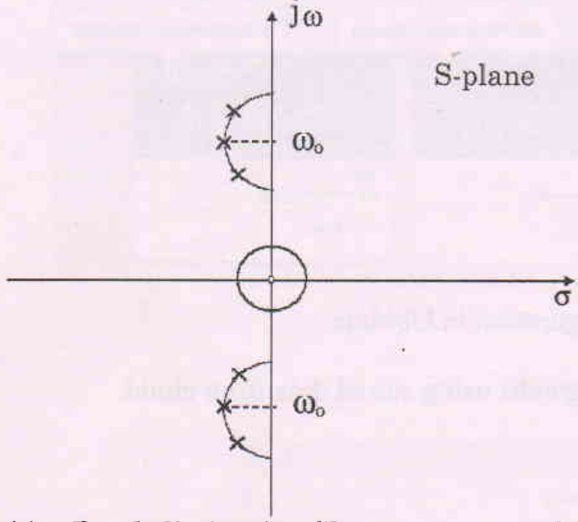
$$20\log_{10} m = -\cancel{20 \times r \log_{10}\omega}^{\,0} + 20\log_{10} K \to \boxed{m = \frac{K}{\omega^r}}$$

$$20db = 20\log_{10} 10 = 20\log_{10} K$$

$\boxed{K=10}$

$$\boxed{T(s) = \frac{10^8 (s+.1)^3}{(s+10)^2(s+100)}} \to \text{option c}$$

3. The given figure shows the pole zero pattern of a filter in the S-plane. The Filter in question is a



(a) Band elimination filter
(b) Band pass filter
(c) Low Pass Filter
(d) High Pass Filter

$$H(s) = \frac{s}{(s^2+a^2)(s^2+b^2)(s^2+c^2)}$$

$$H(j\omega) = \frac{j\omega}{(-\omega^2+a^2)(-\omega^2+b^2)(-\omega^2+c^2)}$$

(i) $\omega = 0$, $H(0) = 0$

(ii) $\omega = \infty$, $H(\infty) = 0$

LPF ×

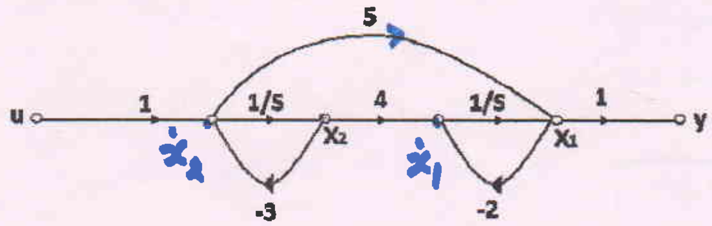
HPF ×

$\boxed{\text{BPF}}$ ✓

BRF → $\omega = 0$, $\omega = \infty$, ×

option (b)

4. From the figure, obtain state equation



(a) $[\dot{X}] = \begin{bmatrix} 0 & -3 \\ -2 & 4 \end{bmatrix}[X] + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

(b) $[\dot{X}] = \begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix}[X] + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$

(c) $[\dot{X}] = \begin{bmatrix} 0 & -3 \\ -2 & 4 \end{bmatrix}[X] + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$

(d) $[\dot{X}] = \begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix}[X] + \begin{bmatrix} 1 \\ 0 \end{bmatrix}u$

$$\dot{x}_1 = -2x_1 + 4x_2 + 0 \cdot u$$

$$\dot{x}_2 = 0 \cdot x_1 - 3x_2 + 1 \cdot u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 4 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

option (b)

5. Laplace transform of $e^{-at}f(t)$ is

(a) $F(s)e^{at}$ (b) $F(s-a)$ (c) $F(s+a)$ (d) $\frac{F(s)}{s}+a$

$f(t) \longrightarrow F(s)$

$e^{-at} f(t) \longrightarrow \underline{F(s+a)}$

option $\underline{C}$

6. System has some poles lying on imaginary axis is

(a) Unconditionally stable (b) Conditionally stable

(c) Unstable (d) Marginally stable

(i) Poles lies on Img axis → marginal.

(ii) Poles " " → multiple → unstable

option (d)

7. The open-loop DC gain of a unity negative feedback system with closed loop transfer function $(S+4)/(S2+7S+13)$ is

(a) 4/13 (b) 4

(c) 4/9 (d) 13

$$\frac{G(s)}{1+G(s)} = \frac{s+4}{s^2+7s+13} ; \frac{N}{D}$$

OLTF

$$G(s) = \frac{N}{D-N} = \frac{s+4}{s^2+7s+13-s-4}$$

$$G(s) = \frac{s+4}{s^2+6s+9}$$

$$G(j\omega) = \frac{j\omega+4}{-\omega^2+j6\omega+9}$$

DC Gain → $f=0,\ \omega=0$

$$\boxed{G(j0) = \frac{4}{9}}$$

option (c)

8. The unit impulse response of a system is $h(t) = e^{-t}$, $t \geq 0$. For this system, the steady-state value of the output for unit step input is equal to

(a) -1 (b) 0

(c) 1 (d) ∞

$h(t) = e^{-t} u(t)$, $\lim_{t \to \infty} y(t) = ?$

$x(t) = u(t)$

$y(t) = x(t) * h(t)$

$Y(s) = X(s) \cdot H(s)$

$Y(s) = \frac{1}{s} \cdot \frac{1}{s+1}$

$\lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} s \cdot \frac{1}{s(s+1)}$

$= 1$ option (c)

9. A system has fourteen poles and two zeros. Its high frequency asymptote in its magnitude plot having a slope of

(a) – 40 dB/decade (b) – 240 dB/decade

(c) – 280 dB/decade (d) – 320 dB/decade

$P = 14$

$Z = 2.$

Final slope in BP $= -20\frac{db}{dec} \times (P-Z)$

$= -20 \times 12$

$= -240$ db/dec

option (b)

10. Consider a unity feedback system having an open loop transfer function

$$G(j\omega) = \frac{k}{j\omega(j0.2\omega+1)(j0.05\omega+1)}$$

Find open loop gain (k) with gain margin of 20 dB

(a) 5.2 (b) 2.5 ✓ (c) 0.1 (d) 2.25

$GM = 20\,db$

$$G(jw) = \frac{k}{jw(j.2w+1)(j.05w+1)}$$

$$GM = -20\log|G(jw)|_{w = w_{pc}}$$

$w_{pc} \rightarrow \phi = -180^\circ$

$$\phi = -90 + \tan^{-1}.2w_{pc} + \tan^{-1}.05w_{pc} = -180^\circ + 90$$

$$\tan^{-1}.2w_{pc} + \tan^{-1}.05w_{pc} = 90^\circ$$

$$\tan^{-1}\frac{.2w_{pc} + .05w_{pc}}{1 - .2w_{pc} \times .05w_{pc}} = 90 = \infty = \frac{1}{0}$$

$$1 - .010w_{pc}^2 = 0 \Rightarrow w_{pc}^2 = \frac{100}{.01},\ w_{pc} = 10.$$

$$GM = -20\log\left|\frac{k}{j10(j2+1)(j.5+1)}\right| = -20\log\frac{k}{25}$$

-20*log(k/25)

$$-20\log\frac{k}{25} = 20$$

$$20\log_{10}\frac{25}{k} = 20\log_{10}10$$

$$\frac{25}{k} = 10$$

$$\boxed{k = \frac{25}{10} = 2.5}$$

option (b)

11. The open loop transfer function of a unity feedback system is

$$G(S) = \frac{K}{S(S^2+S+2)(S+3)}$$

The range of K for which the system is stable is

(a) $\frac{21}{4} > K > 0$ (b) $13 > K > 0$ (c) $\frac{21}{44} < K < \infty$ (d) $-6 < K < \infty$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s^2+s+2)(s+3)} = 0$$

$$(s^3+s^2+2s)(s+3) + K = 0$$

$$s^4 + s^3 + 2s^2 + 3s^3 + 3s^2 + 6s + K = 0$$

$$s^4 + 4s^3 + 5s^2 + 6s + K = 0$$

s^4	1	5	K
s^3	4	6	
s^2	$\frac{7}{2}$	K	
s^1	$\frac{21-4K}{7/2}$		
s^0	K		

$\boxed{0 < K < \frac{21}{4}}$

Option (a)

$\frac{21-4K}{7/2} > 0 \rightarrow 21 - 4K > 0$

$-4K > -21$

$4K < 21$

$\boxed{K < \frac{21}{4}}$

$\boxed{K > 0}$